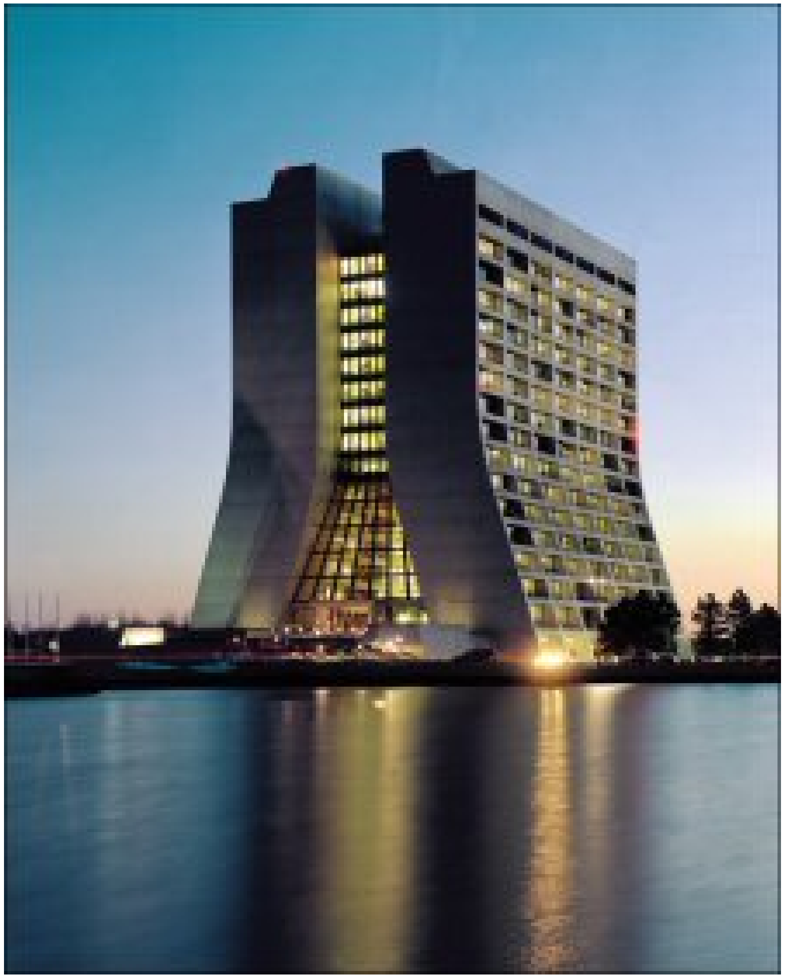
Tests of the Electroweak Theory

- History/introduction
- Weak charged current
- QED
- Weak neutral current
- Precision tests
- Rare processes
- CP violation and B decays
- Neutrino mass



References (mainly weak neutral current/Z-pole)

- **E.D. Commins and P.H. Bucksbaum,** *Weak Interactions of Leptons and Quarks*, **(Cambridge Univ. Press, Cambridge, 1983)**
- **P. Renton,** *Electroweak Interactions*, **(Cambridge Univ. Press, Cambridge, 1990)**
- **P. Langacker, M.-X. Luo and A. K. Mann, High precision electroweak experiments, Rev. Mod. Phys. 64, 87 (1992)**
- *Precision Tests of the Standard Electroweak Model*, **ed. P. Langacker (World Scientific, Singapore, 1995)**
- **S. Eidelman** *et al.* **[Particle Data Group], Phys. Lett. B 592, 1 (2004)** (Electroweak review, J. Erler and PL)
- **LEP Electroweak Working Group:** `http://www.cern.ch/LEPEWWG/`

Weak-Electromagnetic Interference

- **Low energy: Z exchange much smaller than Coulomb, but observe $V-A$ (parity-violating) and $A-A$ (parity conserving) effects**
- **High energy: γ and Z may be comparable (propagator effects)**
- **Observables**
  - **Polarization (charge) asymmetries in $eD \rightarrow eX$ (SLAC), $\mu C \rightarrow \mu X$ (CERN); e^-e^- Møller (SLAC); low energy elastic or quasi-elastic (Mainz, Bates, CEBAF)**
  - **Atomic parity violation in Cs (Boulder, Paris) and other atoms**
  - **Cross sections and FB asymmetries in $e^+e^- \rightarrow \ell\bar{\ell}, q\bar{q}, b\bar{b}$ (SPEAR, PEP, DORIS, TRISTAN, LEP II)**
  - **FB asymmetries in $\bar{p}p \rightarrow e^+e^-$ (CDF, D0)**

- **Parity-violating e-hadron**

$$L^{eq} = \frac{G_F}{\sqrt{2}} \sum_i \left[C_{1i}\, \bar{e}\, \gamma_\mu\, \gamma^5\, e \quad \bar{q}_i\, \gamma^\mu\, q_i + C_{2i}\, \bar{e}\, \gamma_\mu\, e \quad \bar{q}_i\, \gamma^\mu\, \gamma^5\, q_i \right]$$

- **Standard model** (leading: $\rho \sim 1 + \rho_t,\ \kappa \sim 1,\ \lambda \sim 0$)

$$\begin{aligned}
C_{1u} &\sim -\frac{1}{2} + \frac{4}{3}\sin^2\theta_W \rightarrow \rho'_{eq}\left(-\frac{1}{2} + \frac{4}{3}\hat{\kappa}'_{eq}\ \hat{s}^2_Z\right) + \lambda_{1u} \\
C_{1d} &\sim \frac{1}{2} - \frac{2}{3}\sin^2\theta_W \rightarrow \rho'_{eq}\left(\frac{1}{2} - \frac{2}{3}\hat{\kappa}'_{eq}\ \hat{s}^2_Z\right) + \lambda_{1d} \\
C_{2u} &\sim -\frac{1}{2} + 2\sin^2\theta_W \rightarrow \rho_{eq}\left(-\frac{1}{2} + 2\hat{\kappa}_{eq}\ \hat{s}^2_Z\right) + \lambda_{2u} \\
C_{2d} &\sim \frac{1}{2} - 2\sin^2\theta_W \rightarrow \rho_{eq}\left(\frac{1}{2} - 2\hat{\kappa}_{eq}\ \hat{s}^2_Z\right) + \lambda_{2d}
\end{aligned}$$

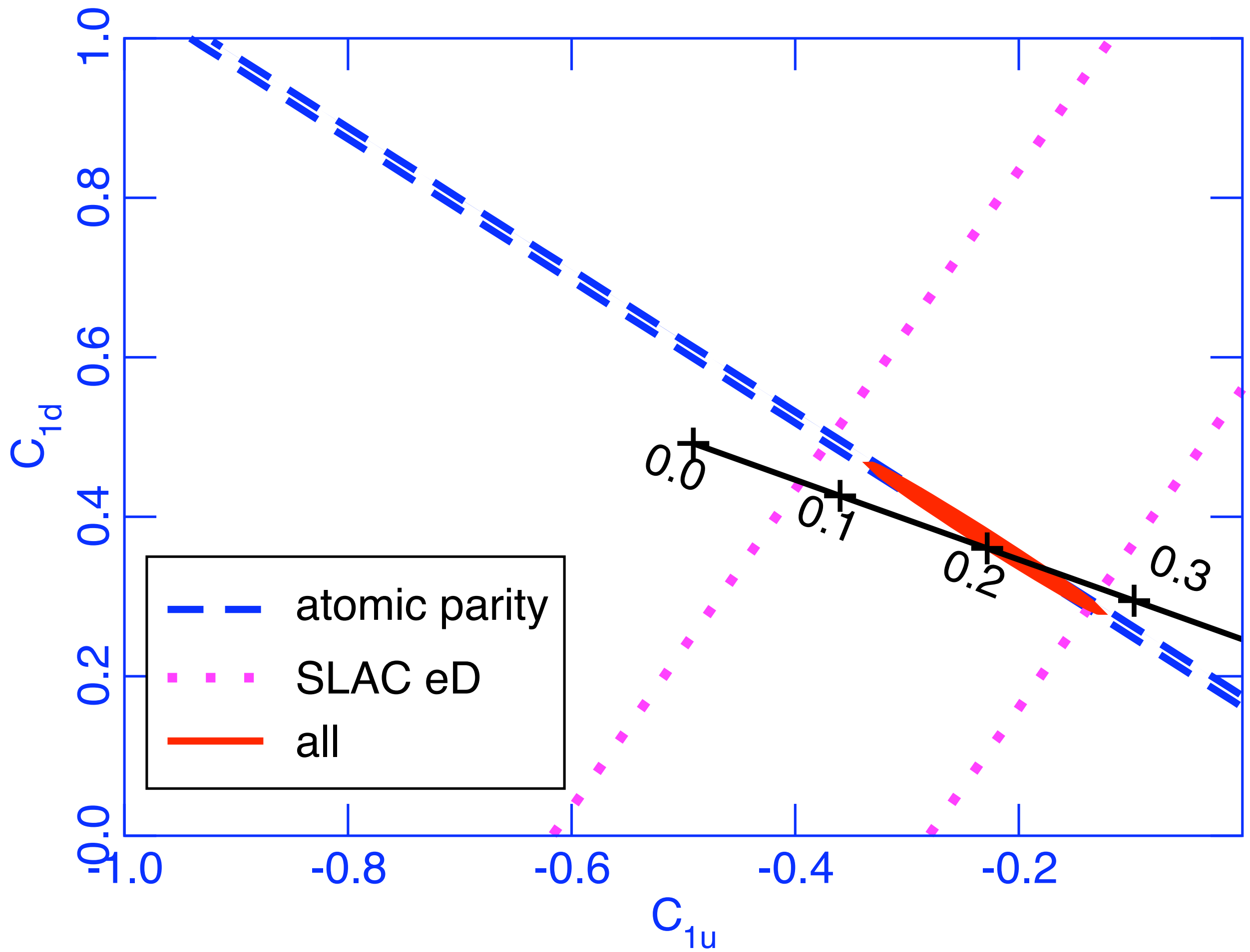
- **Atomic parity violation**
  - **Axial e^-, vector nucleon currents lead to potential**

$$V(\vec{r}_e) \sim \frac{G_F}{4\sqrt{2}} Q_W \delta^3(\vec{r}_e) \frac{\vec{\sigma}_e \cdot \vec{v}_e}{c} + \text{ HC}$$

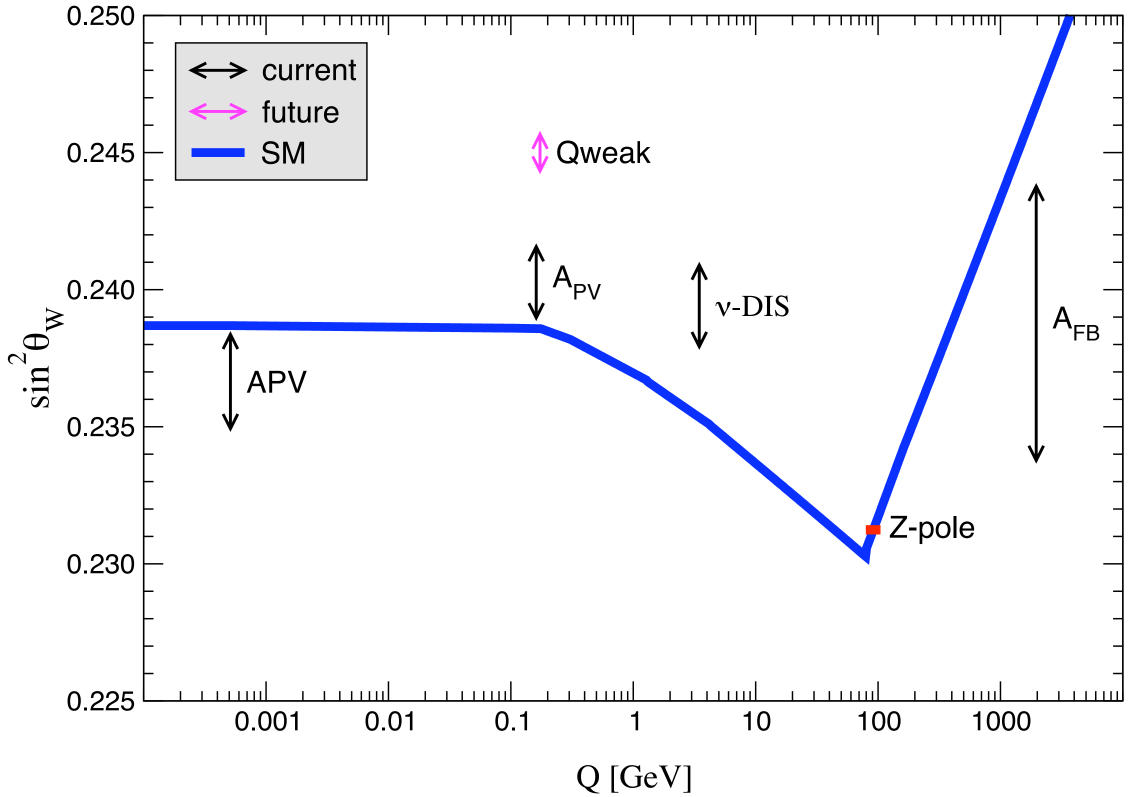
  - **Weak charge**

$$\begin{aligned} Q_W &= -2\left[C_{1u}\,(2Z+N) + C_{1d}(Z+2N)\right] \\ &\approx Z(1-4\sin^2\theta_W) - N \end{aligned}$$

  - **Measure in $6S-7S$ transition ($S-P$ wave mixing)**
  - **Cs is very simple atom; radiative corrections now under control**



1.0
0.8
0.6
0.4
0.2
0.0
C_{1d}
-1.0
-0.8
-0.6
-0.4
-0.2
C_{1u}
0.0
0.1
0.2
0.3
atomic parity
SLAC eD
all



(Running $\hat{s}^2_Z$ in $\overline{\mathrm{MS}}$ scheme)

- **SLAC E158 Polarized Møller Asymmetry**
  - $e^- e^-$ **asymmetry,** $P \sim$ **90%**
  - $\sin^2 \theta_W^{eff}(Q) = 0.2397 \pm 0.0013$ **at** $Q^2 = 0.026$ **GeV^2**
- **Future** Q_{WEAK} **(CEBAF): polarized** ep, $\Delta s^2 \sim 0.0006$

Quantity	Experimental Value	SM	Correlation		
$\epsilon_L(u)$	0.326 ± 0.013	$0.3459(1)$			
$\epsilon_L(d)$	-0.441 ± 0.010	$-0.4291(1)$	non-		
$\epsilon_R(u)$	$-0.175\ ^{+0.013}_{-0.004}$	$-0.1550(1)$	Gaussian		
$\epsilon_R(d)$	$-0.022\ ^{+0.072}_{-0.047}$	0.0776			
g_L^2	0.3005 ± 0.0012	$0.3038(2)$	-0.11	-0.21	-0.01
g_R^2	0.0311 ± 0.0010	0.0301		-0.02	-0.03
θ_L	2.51 ± 0.033	$2.4631(1)$			0.26
θ_R	$4.59\ ^{+0.41}_{-0.28}$	5.1765			
$g_V^{\nu e}$	-0.040 ± 0.015	$-0.0396(3)$			-0.05
$g_A^{\nu e}$	-0.507 ± 0.014	$-0.5064(1)$			
$C_{1u} + C_{1d}$	0.147 ± 0.004	$0.1529(1)$	0.95	-0.75	-0.10
$C_{1u} - C_{1d}$	-0.604 ± 0.066	$-0.5297(4)$		-0.79	-0.10
$C_{2u} + C_{2d}$	0.72 ± 0.89	-0.0095			-0.11
$C_{2u} - C_{2d}$	-0.071 ± 0.044	$-0.0621(6)$			

- $e^+e^- \to \ell^+\ell^-$, $\ell = \mu$ or τ (t-channel for Bhabha, $e^+e^- \to e^+e^-$)
  - Rate: $R =$ rate relative to pure QED
  - Forward-backward asymmetry: $A_{FB} \equiv \frac{\sigma_F - \sigma_B}{\sigma_F + \sigma_B}$

$$R = F_1 \qquad\qquad A_{FB} = 3F_2/4F_1$$

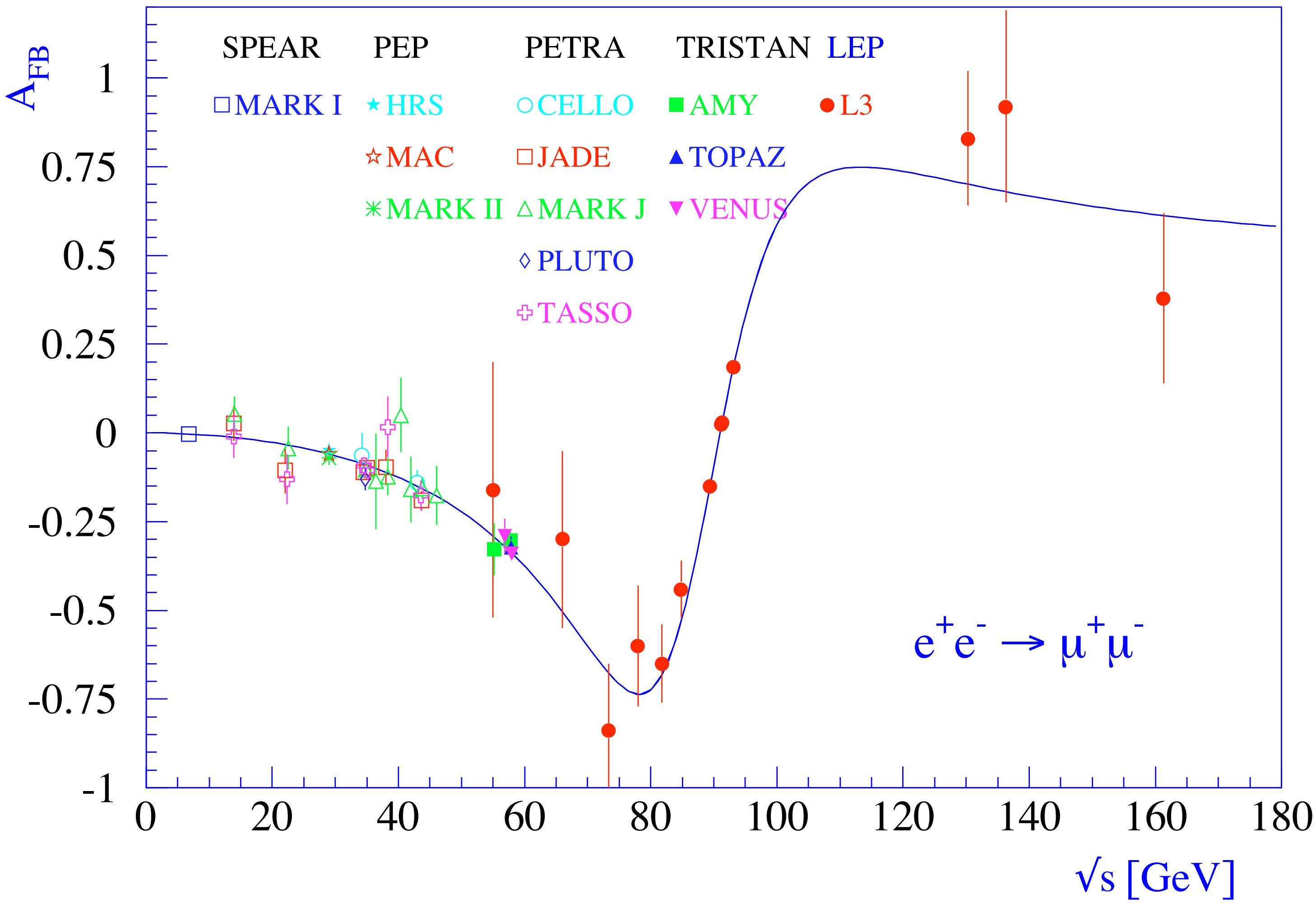
where

$$\begin{aligned} F_1 &= 1 - 2\chi_0 \, g_V^e \, g_V^\ell \, \cos\delta_R + \chi_0^2 \left(g_V^{e2} + g_A^{e2}\right)\left(g_V^{\ell 2} + g_A^{\ell 2}\right) \\ F_2 &= -2\chi_0 \, g_A^e \, g_A^\ell \, \cos\delta_R + 4\chi_0^2 \, g_A^e \, g_A^\ell \, g_V^e \, g_V^\ell \end{aligned}$$

with

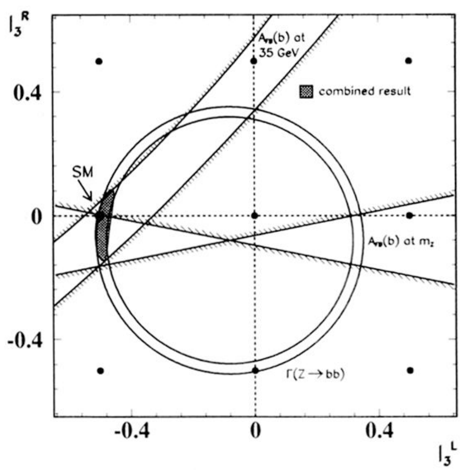
$$\chi_0 = \frac{G_F}{2\sqrt{2}\pi\alpha} \frac{sM_Z^2}{\left[(M_Z^2 - s)^2 + M_Z^2\Gamma_Z^2\right]^{1/2}} \qquad\qquad \tan\delta_R = \frac{M_Z\Gamma_Z}{M_Z^2 - s}$$

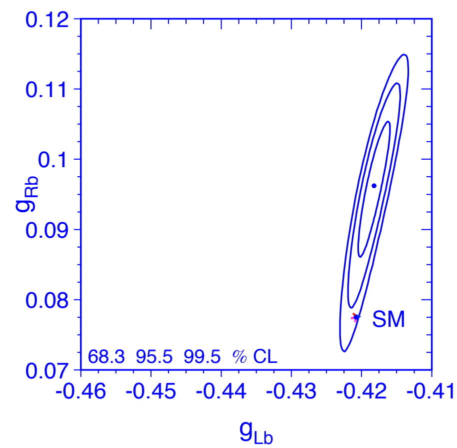
  - SM: $g_A^\ell = -\frac{1}{2} \qquad g_V^\ell = -\frac{1}{2} + 2\sin^2\theta_W$


SPEAR
PEP
PETRA
TRISTAN
LEP
MARK I
HRS
CELLO
AMY
L3
MAC
JADE
TOPAZ
MARK II
MARK J
VENUS
PLUTO
TASSO
A_{FB}
$e^+e^- \to \mu^+\mu^-$
√s [GeV]
1
0.75
0.5
0.25
0
-0.25
-0.5
-0.75
-1
0
20
40
60
80
100
120
140
160
180

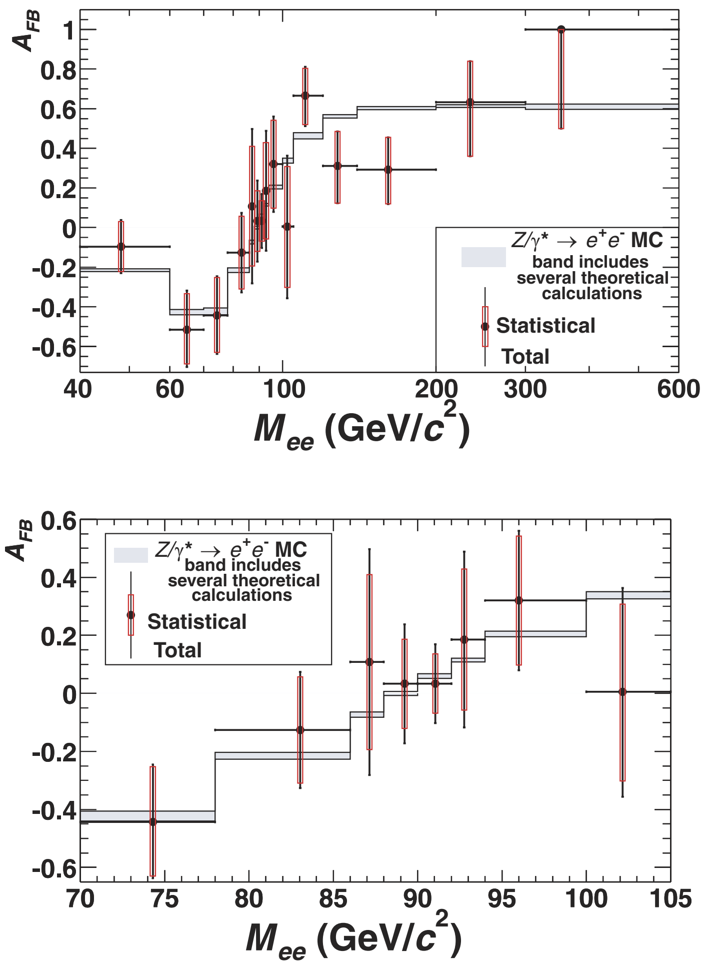
The Weak Interactions of the b

- $e^+e^- \to b\,\bar{b}$ (full strength interaction)
  - Jade (DESY, 1988): A^b_{FB}(35 GeV) $\to t^b_{3L} - t^b_{3R} = -0.54 \pm 0.15$
  - LEP (1992): Γ_b/Γ_{had} and $A^b_{FB}(M_Z)$; LEP + SLC (2005)

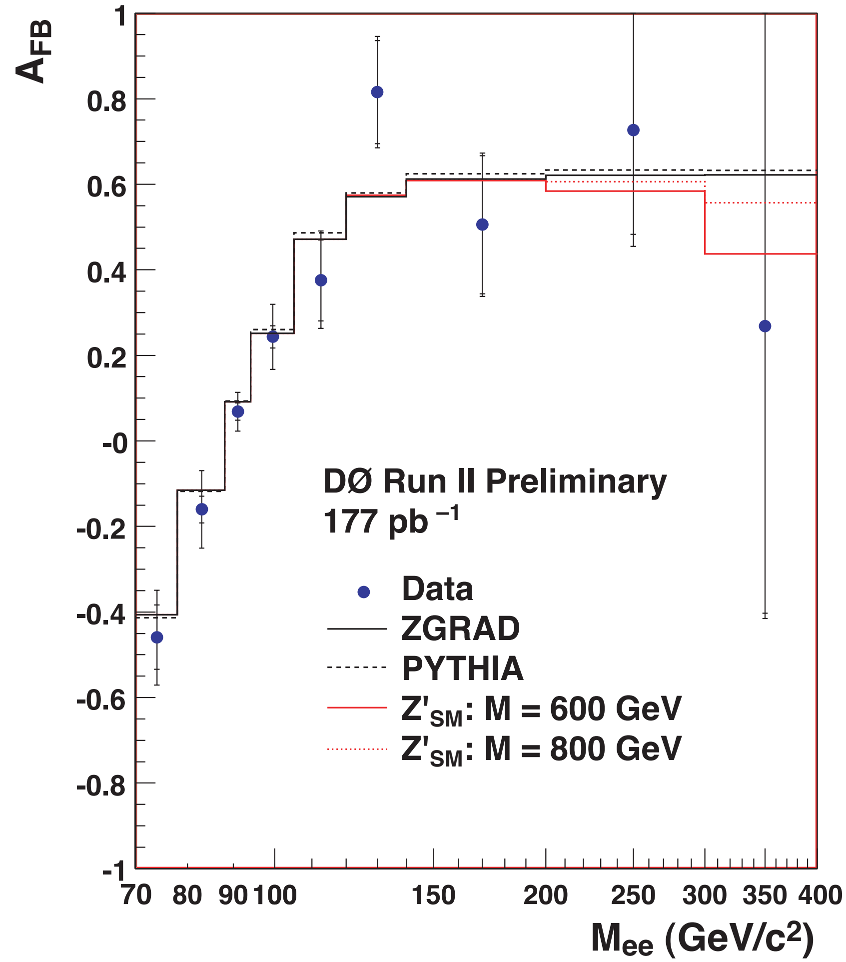




- CLEO (1987): absence of FCNC $B \to l^+l^-X$ (but reduced strength)
- ARGUS (1987), CLEO: $B^0 - \bar{B}^0$ oscillations $\to m_t > 50$ GeV
- Hence, b in left-handed doublet $\to t$ quark must exist (1988) (more general than anomaly cancellation)

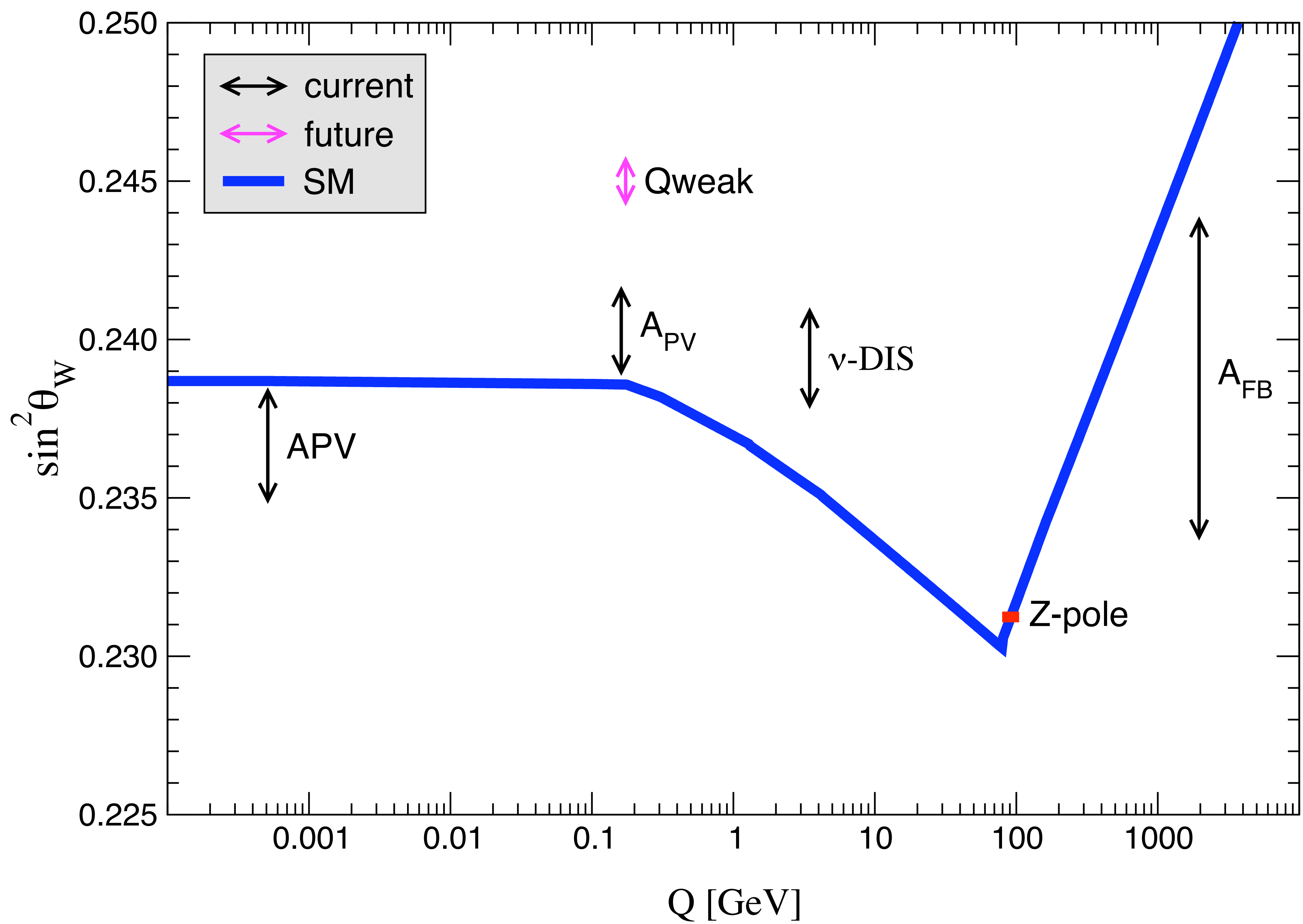


(CDF)



(D0)

Forward-backward asymmetry in $\bar{p}p \to e^+e^-$



(Running $\hat{s}_Z^2$ in $\overline{\mathrm{MS}}$ scheme)

Precision Tests

- **The W and Z Masses and Decays**
- **The Z pole**
- **LEP II**
- **Global fits**
- **Beyond the standard model**

The W and Z Masses and Decays

- On-shell scheme, $s_W^2 \equiv 1 - M_W^2/M_Z^2$

$$M_W = \frac{A_0}{s_W(1-\Delta r)^{1/2}} \qquad\qquad M_Z = \frac{M_W}{c_W}$$

$c_W^2 = 1 - s_W^2$, $A_0 = (\pi\alpha/\sqrt{2}G_F)^{1/2} = 37.2805(2)$ GeV
$\Delta r \rightarrow$ rad. corrections relating α, $\alpha(M_Z)$, G_F, M_W, and M_Z

$$\Delta r \sim \underbrace{1 - \frac{\alpha}{\hat{\alpha}(M_Z)}}_{0.06654(14)} - \underbrace{\frac{\rho_t}{\tan^2\theta_W}}_{\text{artificially large}} + \text{small}$$

$$\rho_t \equiv \frac{3}{8}\frac{G_F m_t^2}{\sqrt{2}\pi^2} = 0.00935\left(\frac{m_t}{172.7\text{ GeV}}\right)^2$$

- **Modified minimal subtraction ($\overline{\mathrm{MS}}$) scheme**

$$M_W = \frac{A_0}{\hat{s}_Z (1 - \Delta \hat{r}_W)^{1/2}} \qquad M_Z = \frac{M_W}{\hat{\rho}^{1/2} \hat{c}_Z}$$

$$\Delta \hat{r}_W \sim \underbrace{1 - \frac{\alpha}{\hat{\alpha}(M_Z)}}_{0.06654(14)} + \text{small}$$

$$\hat{\rho} \sim 1 + \underbrace{\frac{3}{8} \frac{G_F m_t^2}{\sqrt{2} \pi^2}}_{\rho_t \sim 0.00935} + \text{small}$$

- **The W decay width**

$$\Gamma(W^+ \to e^+ \nu_e) = \frac{G_F M_W^3}{6\sqrt{2}\pi} \approx 226.29 \pm 0.16 \text{ MeV}$$

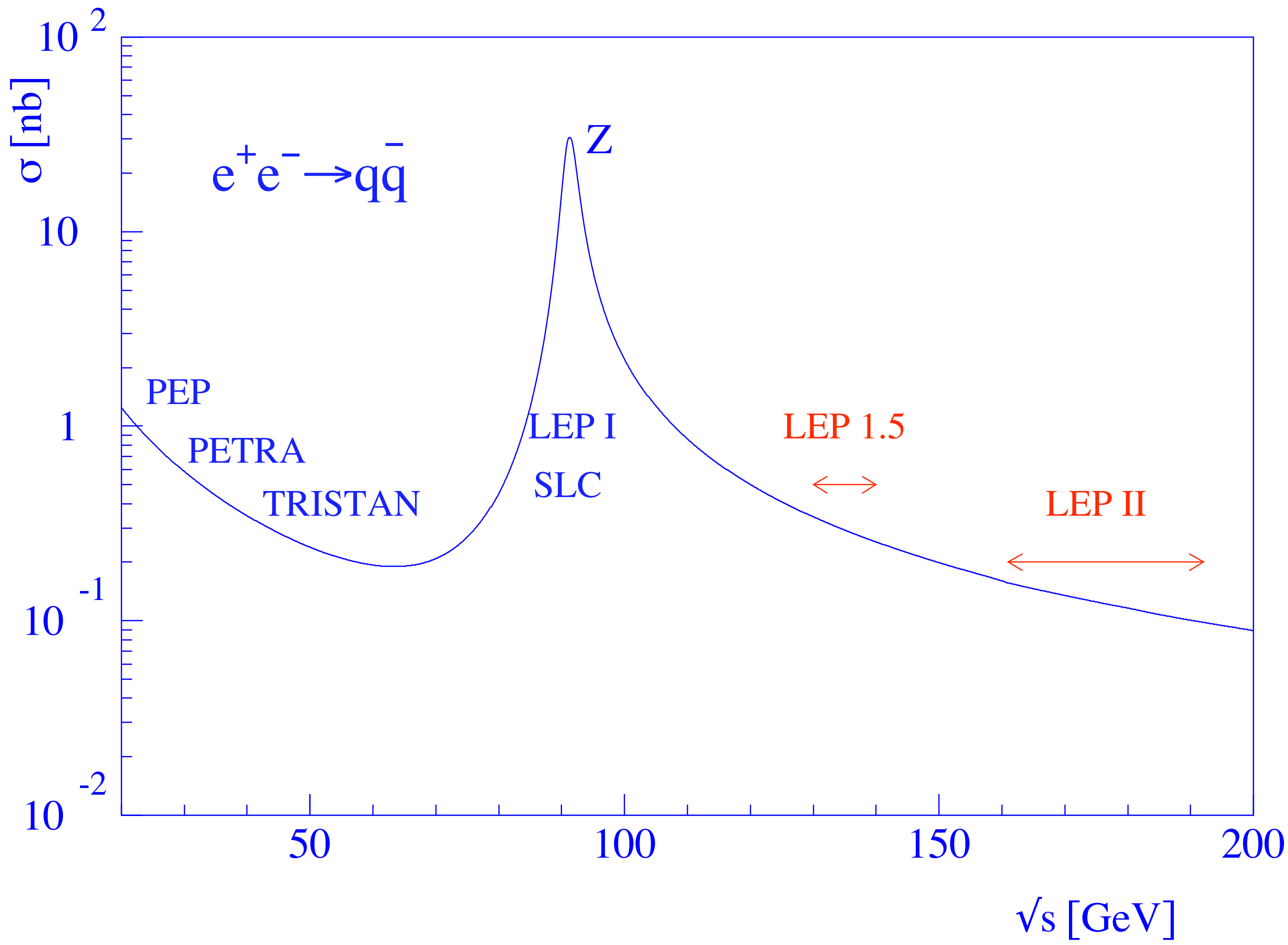
$$\Gamma(W^+ \to u_i \bar{d}_j) = \frac{C G_F M_W^3}{6\sqrt{2}\pi} |V_{ij}|^2 \approx (706.24 \pm 0.49)\, |V_{ij}|^2 \text{ MeV}$$

$$C = \begin{cases} 1, & \text{leptons} \\ \underbrace{3}_{\text{color}} \left(1 + \frac{\alpha_s(M_W)}{\pi} + 1.409 \frac{\alpha_s^2}{\pi^2} - 12.77 \frac{\alpha_s^3}{\pi^3}\right), & \text{quarks} \end{cases}$$

  - Also, QED, mass; $g^2 M_W / 4\sqrt{2} \to G_F M_W^3$ absorbs running α
  - $\Gamma_W \sim 2.0910 \pm 0.0015$ GeV (SM)
  - Experiment (LEP,CDF, D0): $\Gamma_W = 2.138 \pm 0.044$ GeV; $\bar{p}p$ uses

$$\frac{\sigma(\bar{p}p \to W \to \ell\nu_\ell)}{\sigma(\bar{p}p \to Z \to \ell\bar{\ell})} = \underbrace{\frac{\sigma(\bar{p}p \to W)}{\sigma(\bar{p}p \to Z)}}_{\text{theory}} \underbrace{\Gamma(W \to \ell\nu_\ell)}_{\text{theory}} \underbrace{\frac{1}{B(Z \to \ell\bar{\ell})}}_{\text{LEP}} \frac{1}{\Gamma_W}$$

The Z pole



10^2
10
1
10^-1
10^-2
σ [nb]
e+e−→qq̄
Z
PEP
PETRA
TRISTAN
LEP I
SLC
LEP 1.5
LEP II
50
100
150
200
√s [GeV]

The Effective Weak Angle for On-Shell Z

- $Zf\bar{f}$ coupling relevant for on-shell Z

$$\mathcal{L}^{Zf\bar{f}} = -\frac{g}{2\cos\theta_W} Z_\mu \; \bar{f}\gamma^\mu(\bar{g}_{Vf} - \bar{g}_{Af}\gamma^5)f$$

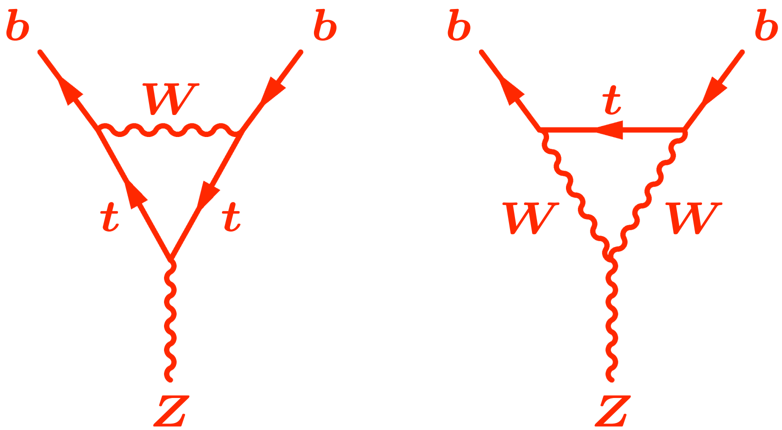
$$\begin{aligned}
\bar{g}_{Af} &= \sqrt{\rho_f}\, t^f_{3L} \; \underbrace{\longrightarrow}_{\text{tree}} \; t^f_{3L} \\
\bar{g}_{Vf} &= \sqrt{\rho_f}\left(t^f_{3L} - 2\kappa_f\, s^2_W q_f\right) \; \underbrace{\longrightarrow}_{\text{tree}} \; t^f_{3L} - 2\sin^2\theta_W\, q_f
\end{aligned}$$

- t^f_{3L} is weak isospin of f ($+\frac{1}{2}$ for (u,ν); $-\frac{1}{2}$ for (d,e^-)); q_f is the electric charge

- ρ_f and κ_f incorporate (f-dependent) electroweak correction (mainly propagator and vertex)

- **On-shell ($f \neq b$):** $\rho_f \sim 1 + \rho_t \qquad \kappa_f \sim 1 + \frac{\rho_t}{\tan^2\theta_W}$

- $\overline{\text{MS}}$ **($f \neq b$):** $\rho_f \to \hat{\rho}_f \sim 1,\ \kappa_f \to \hat{\kappa}_f \sim 1,\ s_W^2 \to \hat{s}_Z^2$
  (e.g., $\hat{\rho}_\ell = 0.9981,\ \hat{\kappa}_\ell = 1.0013$)



  – $\hat{\rho}_b$, $\hat{\kappa}_b$ have quadratic m_t dependence from vertex
  ($\hat{\rho}_b = 0.9870,\ \hat{\kappa}_b = 1.0067$)

- **Effective weak angle: (f-dependent) tree level formula** (except ρ_f)

$$\bar{g}_{Af} = \sqrt{\rho_f}\, t_{3L}^f \qquad \bar{g}_{Vf} = \sqrt{\rho_f}\left(t_{3L}^f - 2\bar{s}_f^2 q_f\right)$$

$$\bar{s}_f^2 = \kappa_f s_W^2 = \hat{\kappa}_f \hat{s}_Z^2 \quad \longrightarrow \quad \bar{s}_\ell^2 = \hat{\kappa}_\ell \hat{s}_Z^2 \sim \hat{s}_Z^2 + 0.00029$$

The LEP/SLC Era

- Z **Pole:** $e^+e^- \to Z \to \ell^+\ell^-, \quad q\bar{q}, \quad \nu\bar{\nu}$
  - **LEP (CERN),** 2×10^7 $Z's$**, unpolarized (ALEPH, DELPHI, L3, OPAL); SLC (SLAC),** 5×10^5, $P_{e^-} \sim$ **75 % (SLD)**

- Z **pole observables**
  - **lineshape:** M_Z, Γ_Z, σ
  - **branching ratios**
    - $e^+e^-, \mu^+\mu^-, \tau^+\tau^-$
    - $q\bar{q}, c\bar{c}, b\bar{b}, s\bar{s}$
    - $\nu\bar{\nu} \Rightarrow N_\nu = 2.984 \pm 0.009$ **if** $m_\nu < M_Z/2$
  - **asymmetries: FB, polarization,** P_τ**, mixed**
  - **lepton family universality**

The Z Lineshape

Basic Observables: $e^+e^- \to f\bar{f}$ $(f = e, \mu, \tau, s, b, c,$ hadrons) $(s = E^2_{CM})$

$$\sigma_f(s) \sim \sigma_f \frac{s\Gamma_Z^2}{(s - M_Z^2)^2 + \frac{s^2\Gamma_Z^2}{M_Z^2}}$$

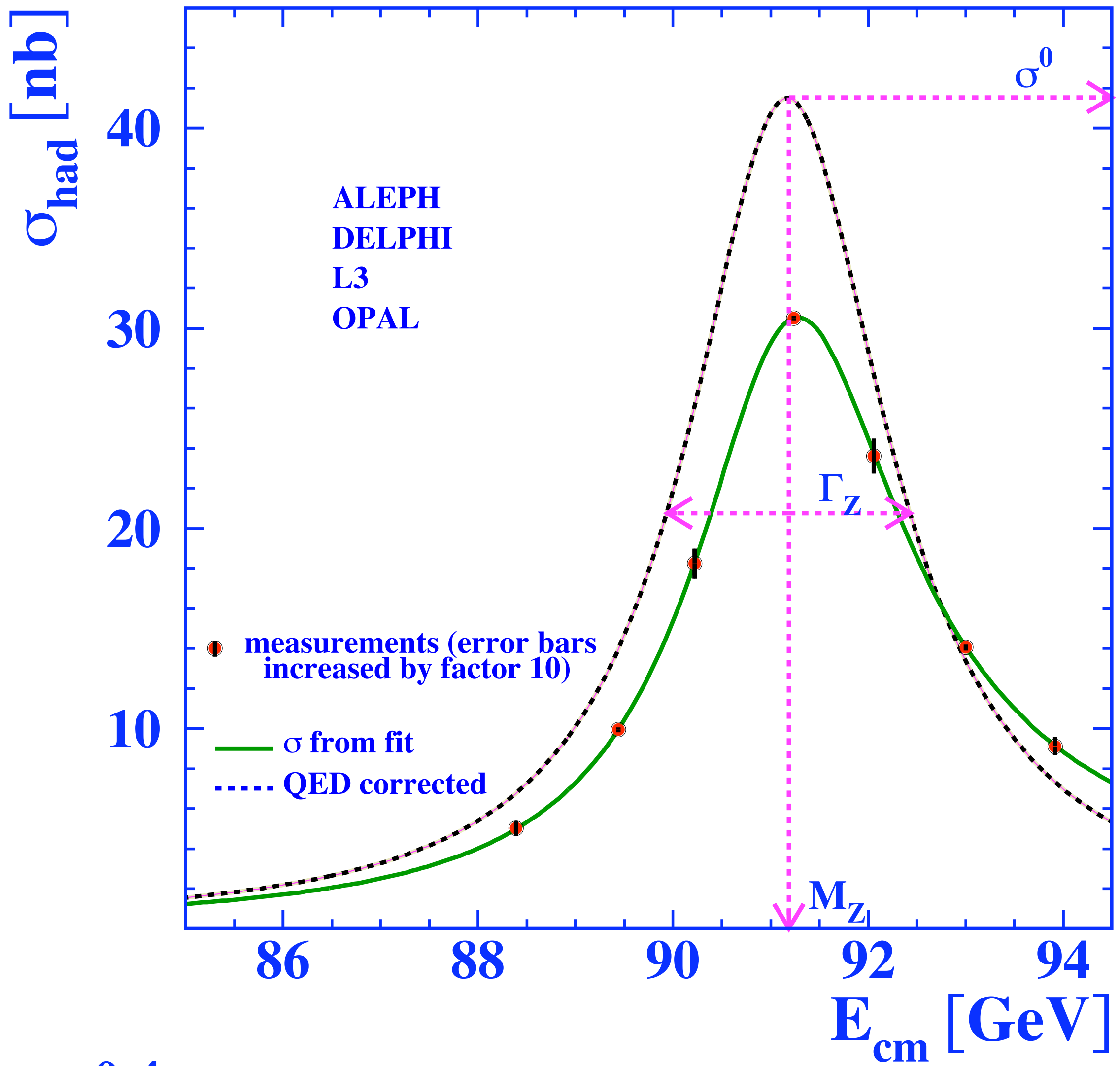
(plus initial state rad. corrections)

M_Z and Γ_Z: from peak position and width

Peak Cross Section:

$$\sigma_f = \frac{12\pi}{M_Z^2} \; \frac{\Gamma(e^+e^-)\Gamma(f\bar{f})}{\Gamma_Z^2}$$

(Z model independent; γ and $\gamma - Z$ int. removed, (usually) assuming S.M.)



(LEPEWWG, `hep-ex/0509008`**)**

- The Z width and partial widths

$$\Gamma(f\bar{f}) \sim \frac{C_f G_F M_Z^3}{6\sqrt{2}\pi} \underbrace{\hat{\rho}}_{\text{only } \overline{\mathrm{MS}}} \left[|\bar{g}_{Vf}|^2 + |\bar{g}_{Af}|^2\right]$$

(plus fermion mass, QED (2 loop), QCD (3 loop), mixed QED-QCD (2 loop) corrections; $C_\ell = 1,\ C_q = 3$)

$$\bar{g}_{Af} = \sqrt{\rho_f}\, t_{3L}^f \qquad \bar{g}_{Vf} = \sqrt{\rho_f}\left(t_{3L}^f - 2\bar{s}_f^2 q_f\right)$$

$$\bar{s}_f^2 = \kappa_f s_W^2 = \hat{\kappa}_f \hat{s}_Z^2$$

- Standard model ($m_t = 172.7(2.9)(0.6)$ GeV, $M_H = 117$ GeV)

$$\Gamma(f\bar{f}) \sim \begin{cases} 300.18 \pm 0.14 \text{ MeV}(u\bar{u}), & 167.21 \pm 0.05 \text{ MeV}(\nu\bar{\nu}) \\ 382.97 \pm 0.14 \text{ MeV}(d\bar{d}), & 83.99 \pm 0.03 \text{ MeV}(e^+e^-) \\ 375.95 \mp 0.10 \text{ MeV}(b\bar{b}) & \end{cases}$$

- **Conventional (weakly correlated) observables:** $M_Z, \Gamma_Z, \sigma_{\text{had}}, R_\ell, R_b, R_c$

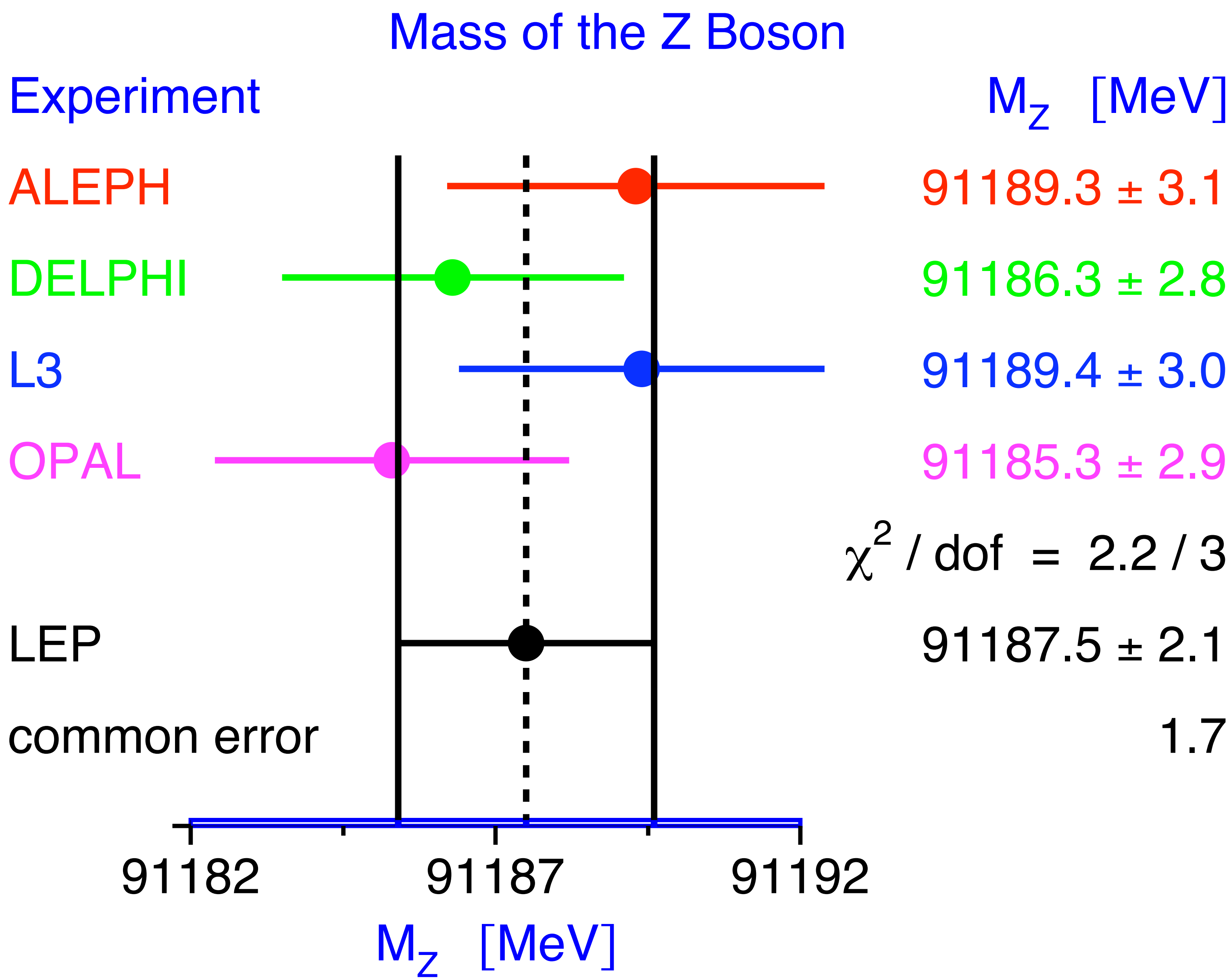
$$
\begin{aligned}
\sigma_{\text{had}} &\equiv \frac{12\pi}{M_Z^2} \frac{\Gamma(e^+e^-)\Gamma(Z \to \text{hadrons})}{\Gamma_Z^2} \\
R_{q_i} &\equiv \frac{\Gamma(q_i\bar{q}_i)}{\Gamma(\text{had})}, \quad q_i = (b, c) \\
R_{\ell_i} &\equiv \frac{\Gamma(\text{had})}{\Gamma(\ell_i\bar{\ell}_i)}, \quad \ell_i = (e, \mu, \tau)
\end{aligned}
$$

(lepton universality test: $R_e = R_\mu = R_\tau \to R_\ell$**)**

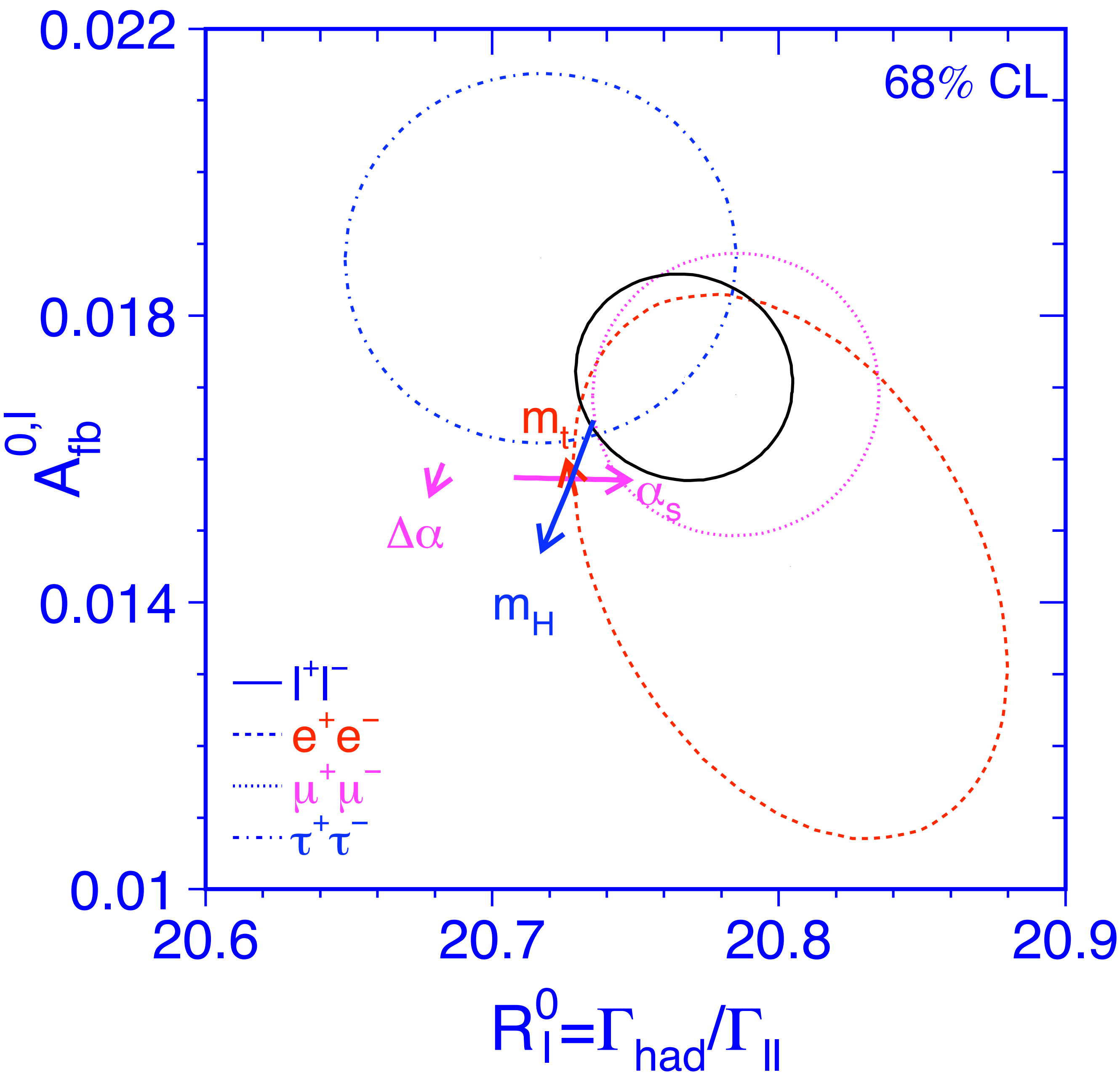
- **Derived**

$$
\Gamma(\text{inv}) = \Gamma_Z - \Gamma(\text{had}) - \sum_i \Gamma(\ell_i\bar{\ell}_i) \equiv N_\nu \Gamma(\nu\bar{\nu})
$$

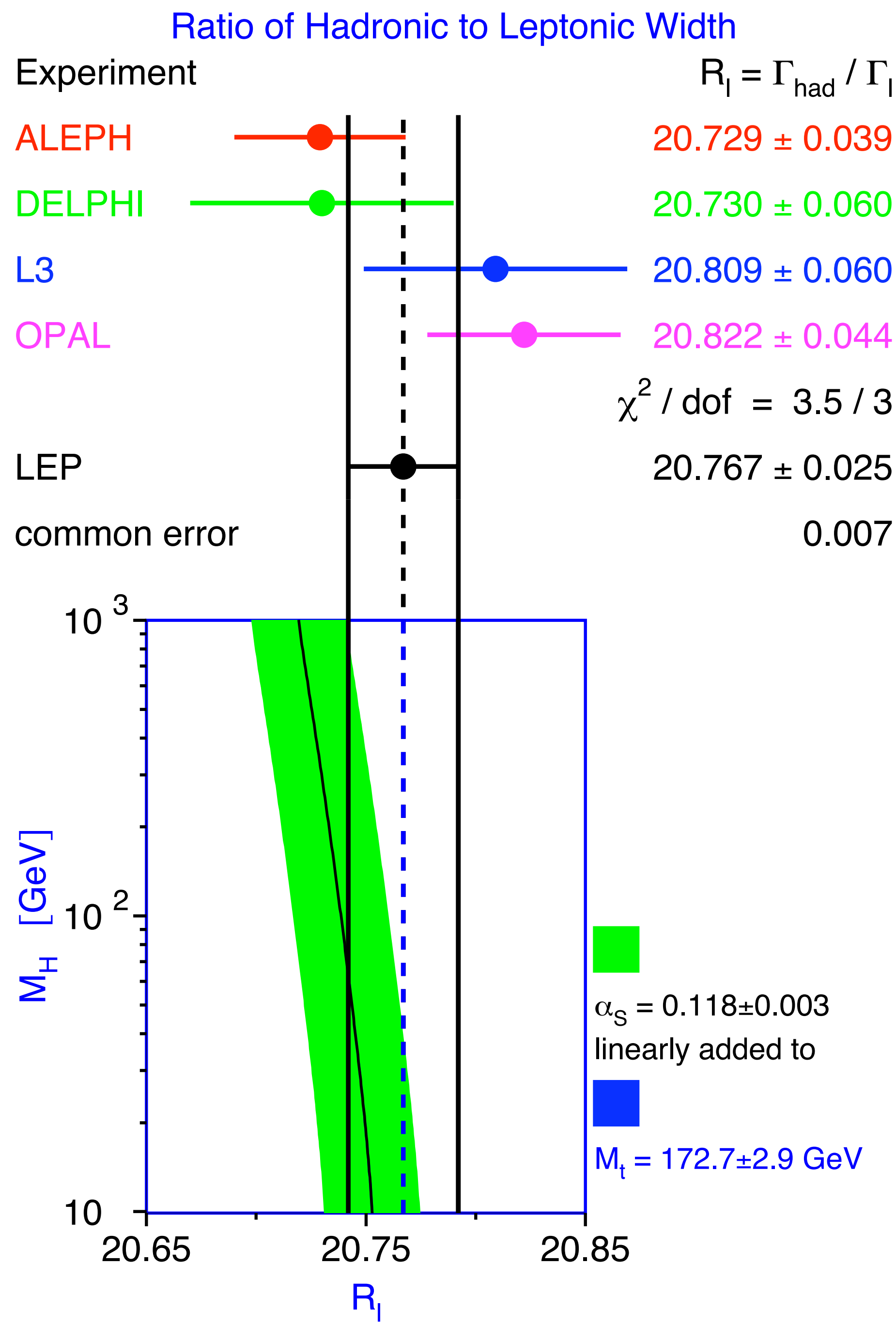
(counts anything invisible in detector)



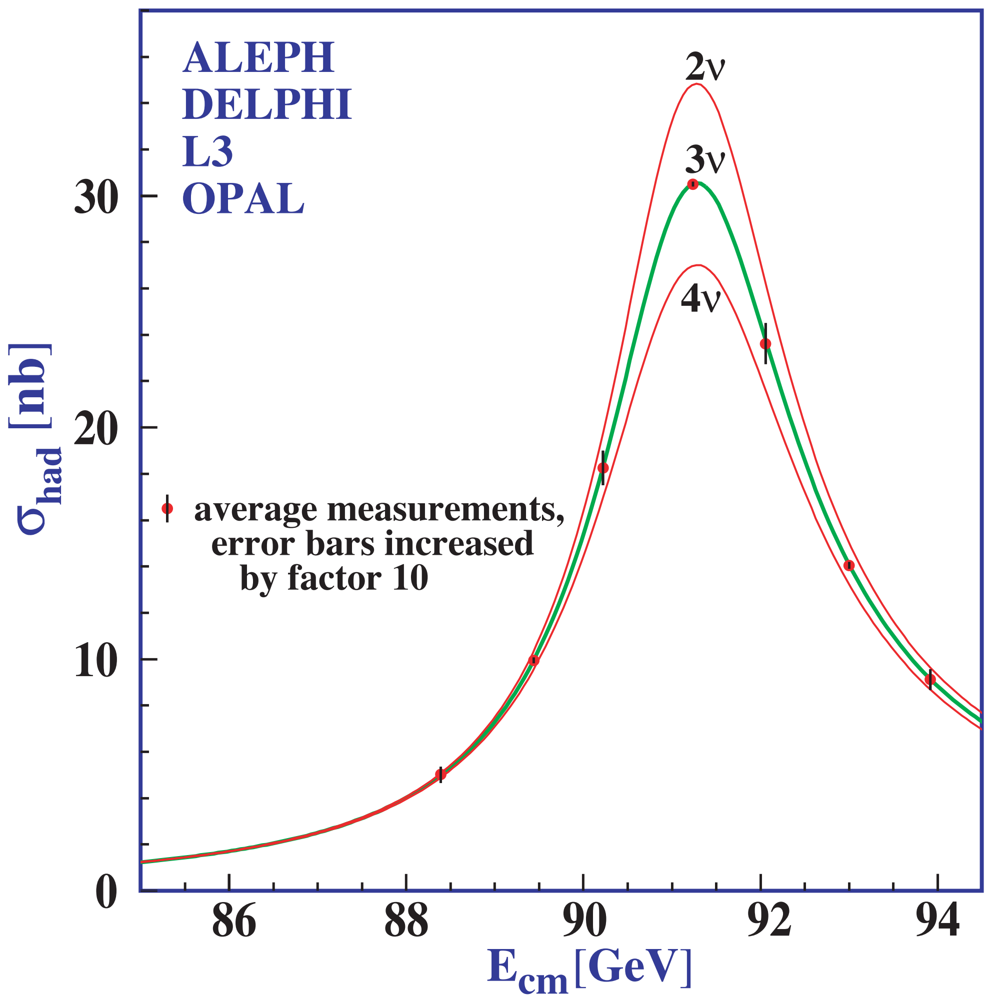
Mass of the Z Boson
Experiment
M_Z [MeV]
ALEPH
91189.3 ± 3.1
DELPHI
91186.3 ± 2.8
L3
91189.4 ± 3.0
OPAL
91185.3 ± 2.9
χ2 / dof = 2.2 / 3
LEP
91187.5 ± 2.1
common error
1.7
91182
91187
91192
M_Z [MeV]



0.022
0.018
0.014
0.01
20.6
20.7
20.8
20.9
68% CL
$A_{fb}^{0,l}$
$R_l^0=\Gamma_{had}/\Gamma_{ll}$
m_t
α_s
$\Delta\alpha$
m_H
l^+l^-
e^+e^-
$\mu^+\mu^-$
$\tau^+\tau^-$



Ratio of Hadronic to Leptonic Width
Experiment
$R_l = \Gamma_{had} / \Gamma_l$
ALEPH
20.729 ± 0.039
DELPHI
20.730 ± 0.060
L3
20.809 ± 0.060
OPAL
20.822 ± 0.044
χ^2 / dof = 3.5 / 3
LEP
20.767 ± 0.025
common error
0.007
10^3
10^2
10
M_H [GeV]
20.65
20.75
20.85
R_l
$\alpha_S = 0.118 \pm 0.003$
linearly added to
$M_t = 172.7 \pm 2.9$ GeV



- $N_\nu = 3 + \Delta N_\nu = 2.984 \pm 0.009$

- $\Delta N_\nu = 1$ for fourth family ν with $m_\nu \lesssim M_Z/2$

- $\Delta N_\nu = \frac{1}{2}$, light $\tilde{\nu}$ in supersymmetry

- $\Delta N_\nu = 2$, Majoron + scalar in triplet model of m_ν with spontaneous L violation

Without lepton universality		Correlations								
$\chi^2/\mathrm{dof} = 32.6/27$		m_Z	Γ_Z	σ^0_{had}	R^0_{e}	R^0_{μ}	R^0_{τ}	$A^{0,\mathrm{e}}_{\mathrm{FB}}$	$A^{0,\mu}_{\mathrm{FB}}$	$A^{0,\tau}_{\mathrm{FB}}$
m_Z [GeV]	91.1876 ± 0.0021	1.000								
Γ_Z [GeV]	2.4952 ± 0.0023	-0.024	1.000							
σ^0_{had} [nb]	41.541 ± 0.037	-0.044	-0.297	1.000						
R^0_{e}	20.804 ± 0.050	0.078	-0.011	0.105	1.000					
R^0_{μ}	20.785 ± 0.033	0.000	0.008	0.131	0.069	1.000				
R^0_{τ}	20.764 ± 0.045	0.002	0.006	0.092	0.046	0.069	1.000			
$A^{0,\mathrm{e}}_{\mathrm{FB}}$	0.0145 ± 0.0025	-0.014	0.007	0.001	-0.371	0.001	0.003	1.000		
$A^{0,\mu}_{\mathrm{FB}}$	0.0169 ± 0.0013	0.046	0.002	0.003	0.020	0.012	0.001	-0.024	1.000	
$A^{0,\tau}_{\mathrm{FB}}$	0.0188 ± 0.0017	0.035	0.001	0.002	0.013	-0.003	0.009	-0.020	0.046	1.000

With lepton universality		Correlations				
$\chi^2/\mathrm{dof} = 36.5/31$		m_Z	Γ_Z	σ^0_{had}	R^0_{ℓ}	$A^{0,\ell}_{\mathrm{FB}}$
m_Z [GeV]	91.1875 ± 0.0021	1.000				
Γ_Z [GeV]	2.4952 ± 0.0023	-0.023	1.000			
σ^0_{had} [nb]	41.540 ± 0.037	-0.045	-0.297	1.000		
R^0_{ℓ}	20.767 ± 0.025	0.033	0.004	0.183	1.000	
$A^{0,\ell}_{\mathrm{FB}}$	0.0171 ± 0.0010	0.055	0.003	0.006	-0.056	1.000

- **Combinations by LEPEWWG,** `hep-ex/0509008`
- **Correlations small but essential**